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1996 J. Phys.: Condens. Matter 8 4515

(http://iopscience.iop.org/0953-8984/8/25/009)

J. Phys.: Condens. Matter **8** (1996) 4515–4527. Printed in the UK

Compensation temperature in a ferrimagnetic Ising system with decorated magnetic and non-magnetic atoms

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Received 5 February 1996

Abstract. The magnetic properties (transition temperature, compensation temperature and magnetization curve) of a decorated Ising system consisting of three kinds of magnetic and non-magnetic atom on the two-dimensional lattice, of which one with spin 1/2 forms a square lattice and the other two (a magnetic atom with a spin $S\,(>1/2)$ and a non-magnetic atom) occupy randomly the middle points of each bond in the square lattice, are investigated within the framework of the effective-field theory with correlations. Particular emphasis is given to the effects of S and crystal-field interaction D in the decorated magnetic atom on them. We find that the compensation temperature in the system may exhibit some interesting behaviours with the variation in S and D as well as the concentration p of the decorated spin-S atoms, such as the possibility of two compensation points induced by the variation in p.

1. Introduction

Ferrimagnetism has been extensively investigated in the past both experimentally and theoretically, since important magnetic materials for applications, such as garnets and ferrites, are ferrimagnetic. Most of the theoretical studies have discussed the fact that there exists only one compensation point in ferrimagnetic systems [1, 2]. Recently, the possibility of many compensation points in a variety of ferrimagnetic systems has been clarified theoretically [3–8].

Decorated Ising spin models, which were originally introduced into the literature by Syozi [9], have been studied many years ago as models exhibiting ferrimagnetism. The arrangement of atoms was like that in the normal spinel. The temperature dependence of the resultant magnetization in the decorated ferrimagnetic models has been investigated and the results have shown some characteristic features in ferrimagnetism [10, 11]. Although the decorated Ising systems with $z \leqslant 4$ (z is the coordination number), such as the square lattice and honeycomb lattice, have been considered as exactly solvable, the decorated models studied so far have been restricted to the situation in which the crystal-field interaction D in the decorated magnetic atoms is considered to be $D = \infty$ [10, 11]. In the limit, some interesting phenomena in ferrimagnetism were obtained for the system with two types of magnetic atom randomly decorated on each bond in the spin-1/2 Ising lattice, such as the possibility of two compensation points [11]. As far as we know, however, no studies have been made on the decorated ferrimagnetic Ising system with a finite (especially negative) crystal-field interaction.

The aim of this work is to investigate the magnetic properties of the decorated ferrimagnetic Ising system consisting of three kinds of magnetic and non-magnetic atom,

0953-8984/96/254515+13$19.50

as depicted in figure 1, in which all open circles are always occupied by a magnetic (or A) atom with $S_A = 1/2$ but any full circle is occupied not by a definite atom but randomly by a magnetic (or B) atom with $S_B = S(> 1/2)$ and crystal-field interaction $D_B = D$ or a non-magnetic (or C) atom. The problem is studied within the framework of the effective-field theory (EFT) [12]. Particular attention is directed to the effects of S, D and the concentration p of the decorated B atoms on the compensation temperature.

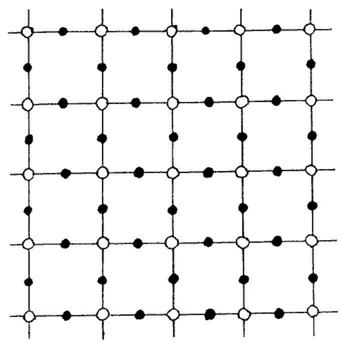

Figure 1. The two-dimensional decorated spin system with three kinds of atom, of which the magnetic A atoms with spin 1/2 form the sublattice L_1 (the square lattice) and the other two (the spin-S B atom and the non-magnetic C atom) are randomly distributed on the sublattice L_2 (full circles).

The outline of this work is as follows. In section 2, we present the general formulation for the ferrimagnetic Ising system with decorated magnetic and non-magnetic atoms in the EFT. An exact formulation for evaluating the compensation temperature of the system is given. In section 3, the magnetic properties (transition temperature, compensation temperature and magnetization curve) of the system on the two-dimensional lattice are examined numerically by changing the values of S, D and p. We find that the compensation temperature may exhibit a variety of interesting behaviours depending on the values of S, D and p.

2. Formulation

We consider a decorated two-sublattice ferrimagnetic Ising system, as depicted in figure 1, where the total number of full circles on the sublattice L_2 is $2N$, $2Np$ points of which are occupied by magnetic B atoms with spin $S\,(> 1/2)$, and $2N(1-p)$ points by non-magnetic C atoms. p is the concentration of B atoms. The Hamiltonian of the system is given by

$$H = J \sum_{(i,m)} \mu_i^z S_m^z \xi_m - J' \sum_{(i,j)} \mu_i^z \mu_j^z - D \sum_m (S_m^z)^2 \xi_m \tag{1}$$

where μ_i^z on the open circles takes the values of $\pm 1/2$ and the spin operator S_m^z randomly distributed on the full circles can take the $(2S+1)$-values allowed for a spin $S\,(S > 1/2)$. The first two summations are carried out only over nearest-neighbour pairs of spins between L_1 and L_2 and on the sublattice L_1. $J(> 0)$ and $J'(> 0)$ are the exchange interactions. ξ_m is the random variable that takes the value of unity or zero, depending on whether the site m on the sublattice L_2 is occupied by a magnetic B atom (with a probability p) or by a non-magnetic C atom (with a probability $1 - p$).

The main problem is now the evaluation of the mean values $\langle \mu_i^z \rangle$ and $\xi_m \langle S_m^z \rangle$ where the angular brackets denote the usual thermal averages. According to the Ising spin identities and the differential operator technique, the quantities can be obtained in a simple fashion, as discussed in the review article in [12]. After performing the random configuration average, the averaged values can be exactly presented as

$$\sigma = \langle\langle \mu_i^z \rangle\rangle_r = \left\langle\!\!\left\langle \left\{ \prod_{\delta=1}^{z} \exp(-a S_{i+\delta}^z \xi_{i+\delta}) \right\} \left\{ \prod_{\delta'=1}^{z} \exp(b \mu_{i+\delta'^z}) \right\} \right\rangle\!\!\right\rangle_r f(x, y)|x = 0, y = 0 \tag{2}$$

and

$$\begin{aligned} m &= \frac{\langle \xi_m \langle S_m^z \rangle\rangle_r}{\langle \xi_m \rangle_r} \\ &= \frac{1}{\langle \xi_m \rangle_r} \left\langle \xi_m \left\langle \left\{ \prod_{\delta''=1}^{2} \exp(-a \mu_{m+\delta''^z}) \right\} \right\rangle\!\!\right\rangle_r F_s(x)|_{x=0} \end{aligned} \tag{3}$$

where $a = J\nabla_x$ and $b = J'\nabla_y$, with $\nabla_x = \partial/\partial x$ and $\nabla_y = \partial/\partial y$ the differential operators. δ, δ' and δ'' denote the nearest neighbours of the central site i or m, and z is the coordination number of sublattice L_1. Here, $\langle \cdots \rangle_r$ denotes the random configuration average for the B atoms on the sublattice L_2. The function $f(x, y)$ in (2) is defined by

$$f(x, y) = \frac{1}{2} \tanh\left(\frac{\beta}{2}(x + y)\right) \tag{4}$$

where $\beta = 1/k_B T$. On the other hand, the function $F_s(x)$ depends on the spin value S of the magnetic B atoms. When the value of S is given by $S = 1$ and 3/2, we have

$$F_s(x) = \frac{2\sinh(\beta x)}{2\cosh(\beta x) + \exp(-D\beta)} \tag{5a}$$

and

$$F_s(x) = \frac{1}{2} \frac{3\sinh(3\beta x/2) + \exp(-2D\beta)\sinh(\beta x/2)}{\cosh(3\beta x/2) + \exp(-2D\beta)\cosh(\beta x/2)}. \tag{5b}$$

The averaged total magnetization M is then given by

$$\frac{M}{N} = \sigma + 2pm. \tag{6}$$

Then, using the identity

$$\exp(-a\mu_i^z) = \cosh(a/2) - 2\mu_i^z \sinh(a/2) \tag{7}$$

the sublattice magnetization m can be evaluated exactly as

$$\begin{aligned} m &= -\frac{1}{\langle \xi_m \rangle_r} \langle \xi_m \langle (\mu_{m+1}^z + \mu_{m+2}^z) \rangle\rangle_r K \\ &= -2\sigma K \end{aligned} \tag{8}$$

with

$$\begin{aligned} K &= 2\sinh(a/2)\cosh(a/2)\; F_s(x)|_{x=0} \\ &= F_s(x). \end{aligned} \tag{9}$$

Thus, the total magnetization M in the system can be expressed exactly as

$$\frac{M}{N} = \sigma[1 - 4pF_s(J)]. \tag{10}$$

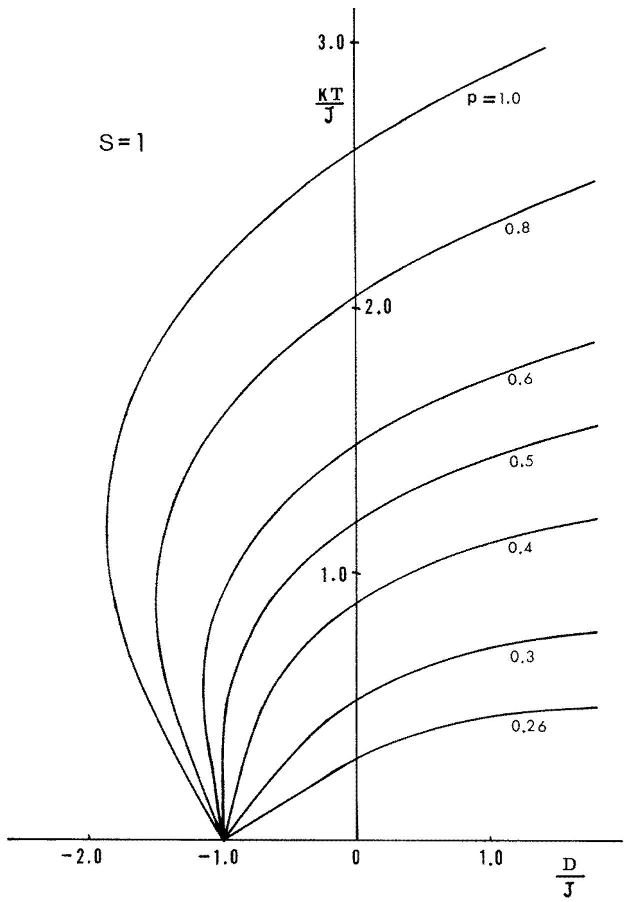

Figure 2. Equation (13), $1 = 4pF_s(J)$, plotted in T–D space, changing the value of p: (a) $S = 1$; (b) $S = 3/2$.

In particular, when $D = \infty$, the function $F_s(x)$ is given by

$$F_s(x) = S\tanh(S\beta x) \tag{11}$$

from which (10) for the pure system with $p = 1$ reduces to

$$\frac{M}{N} = \sigma[1 - 4S\tanh(S\beta J)]. \tag{12}$$

Since $M_0 = N\sigma$ is the spontaneous magnetization of the sublattice L_1, the relation (12) has a form similar to the exact expression (or (3.5)) in [10], although $\tanh(S\beta J)$ in (12) is replaced by $\tanh(2\beta J)$ in [10].

In a ferrimagnet, the sublattice magnetizations σ and m do not have the same sign, and there may be a compensation temperature T_{COMP} at which the total magnetization M reduces to zero even though $\sigma \neq 0$ and $m \neq 0$. From (10), the compensation point in the system seems to be exactly given by

$$1 = 4pF_s(J). \tag{13}$$

Equation (13) is plotted in figures 2(a) and 2(b) as a function of D by selecting the two values of S, namely $S = 1$ and $S = 3/2$, respectively, and changing the value of p.

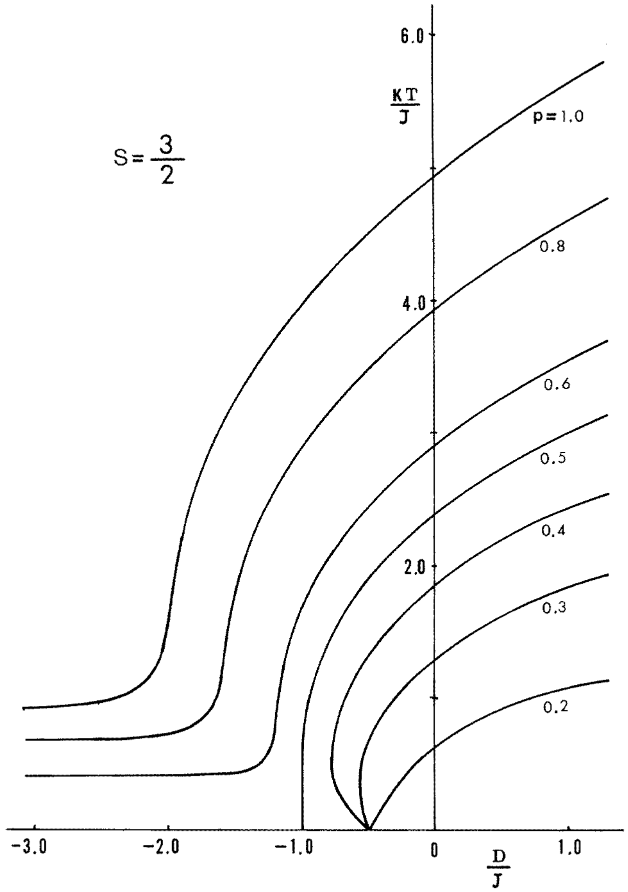

Figure 2. (Continued.)

However, in order that the temperature determined from (13) is actually the compensation temperature, it must be lower than the transition temperature of the system. In other words, σ must be not equal to zero even when equation (13) is satisfied.

The problem is now how to evaluate the sublattice magnetization σ (or (2)) in the system, which can be expressed as

$$\sigma = \left\langle\!\!\left\langle \left\{ \prod_{\delta=1}^{z} [\xi_{i+\delta} \exp(-aS_{i+\delta}^{z}) + 1 - \xi_{i+\delta}] \right\} \left(\prod_{\delta'=1}^{z} \exp(b\mu_{i+\delta'2}) \right) \right\rangle\!\!\right\rangle_r f(x, y)|_{x=0, y=0} \tag{14}$$

where we used $(\xi_i)^n = \xi_i$ (n is an integer). For this, we can use the Van der Waerden identities, such as

$$\exp(-aS_i^z) = 1 - S_i^z \sinh(a) + (S_i^z)^2[\cosh(a) - 1] \qquad \text{for } S = 1 \tag{15}$$

and

$$\exp(-aS_i^z) = A(a) - B(a)S_i^z + C(a)(S_i^z)^2 - D(a)(S_i^z)^3 \qquad \text{for } S = 3/2 \tag{16}$$

with

$$\begin{aligned} A(a) &= \tfrac{1}{8}[9\cosh(a/2) - \cosh(3a/2)] \\ B(a) &= \tfrac{1}{27}[27\sinh(a/2) - \sinh(3a/2)] \\ C(a) &= \tfrac{1}{2}[\cosh(3a/2) - \cosh(a/2)] \\ D(a) &= \tfrac{1}{3}[\sinh(3a/2) - 3\sinh(a/2)] \end{aligned} \tag{17}$$

as well as (7).

Even if we use the Van der Waerden identity, it is generally impossible to calculate the sublattice magnetization σ exactly, although the systems with $D = \infty$ in [10, 11] are exceptional cases. That is to say, if we try to treat exactly all the spin–spin correlations appearing in the sublattice magnetization through the expansion of (14) and to perform properly the random configuration average, the problem becomes mathematically untractable. As discussed in the previous work, therefore, the decoupling approximation, or

$$\langle\langle x_j x_k (x_l)^2 \ldots \mu_m^z \mu_n^z \rangle\rangle_r \sim \langle\langle x_j \rangle\rangle_r \langle\langle x_k \rangle\rangle_r \langle\langle (x_l)^2 \rangle\rangle_r \ldots \langle\langle \mu_m^z \rangle\rangle_r \langle\langle \mu_n^z \rangle\rangle_r \tag{18}$$

with $j \neq k \neq l \neq \cdots \neq m \neq n$ and $x_i = \xi_j S_j^z$, has been used. The approximation (or the EFT) corresponds essentially to the Zernike approximation [12–14]. The approximation has been successfully applied to a great number of disordered systems [12].

Within the framework of the EFT, the sublattice magnetization σ can be written in a compact form as follows:

$$\begin{aligned} \sigma = {} & [p\{1 - m\sinh(a) + q\{\cosh(a) - 1\}\} + 1 - p]^z \\ & \times[\cosh(b/2) + 2\sigma\sinh(b/2)]^z f(x, y)|_{x=0, y=0} \qquad \text{for } S = 1 \end{aligned} \tag{19}$$

and

$$\begin{aligned} \sigma = {} & [p\{A(a) - B(a)m + C(a)q - D(a)r\} + 1 - p]^z \\ & \times[\cosh(b/2) + 2\sigma\sinh(b/2)]^z f(x, y)|_{x=0, y=0} \qquad \text{for } S = 3/2 \end{aligned} \tag{20}$$

where the parameters q and r are defined by

$$q = \frac{\langle \xi_m \langle (S_m^z)^2 \rangle\rangle_r}{\langle \xi_m \rangle_r} \qquad r = \frac{\langle \xi_m \langle (S_m^z)^3 \rangle\rangle_r}{\langle \xi_m \rangle_r}. \tag{21}$$

Within the EFT, we can easily obtain, in the same way as σ and m,

$$q = Q_1 + 4\sigma^2 Q_2 \tag{22}$$

$$r = -2\sigma R \tag{23}$$

with

$$\begin{aligned} Q_1 &= \cosh^2(a/2)\ G_s(x)\ |_{x=0} \\ &= \tfrac{1}{2}[G_s(J) - G_s(0)] \\ Q_2 &= \sinh^2(a/2)\ G_s(x)|_{x=0} \\ &= \tfrac{1}{2}[G_s(J) - G_s(0)] \\ R\ &= 2\ \cosh(a/2)\ \sinh(a/2)\ R_s(x)\ |_{x=0} \\ &= R_s(j) \end{aligned} \tag{24}$$

where the functions $G_s(x)$ and $R_s(x)$ depend on the value of S and are given in the appendix.

3. The two-dimensional decorated system

In this section, let us examine the magnetic properties of the two-dimensional decorated ferrimagnetic system with $z = 4$ depicted in figure 1, where the sublattice L_1 forms the square lattice, on the basis of the formulation given in section 2. Then, let us study them separately by taking the two values of S, namely $S = 1$ and $S = 3/2$, in sections 3.1 and 3.2.

3.1. The decorated system with $S = 1$

Substituting $z = 4$, (8) and (22) into (19) and expanding the right-hand side of (19), we obtain the polynomial equation for σ with odd powers:

$$\sigma = a\sigma + b\sigma^3 + \cdots \tag{25}$$

where the coefficients $a, b, \ldots$ can be easily derived from (19), e.g.

$$a = 8p(KA_1 + A_2) \tag{26}$$

with

$$\begin{aligned} A_1 &= \sinh(a)[1 + pQ_1\{\cosh(a) - 1\}]^3 \cosh^4(b/2) \qquad f(x, y)|_{x=0, y=0} \\ A_2 &= [1 + pQ_1\{\cosh(a) - 1\}]^4 \sinh(b/2)\cosh^3(b/2) \qquad f(x, y)|_{x=0, y=0}. \end{aligned} \tag{27}$$

When the temperature is higher than the transition temperature, the whole system is demagnetized. The demagnetization of the system may be realized continuously or discontinuously at the transition temperature. For the second-order transition, the transition temperature T_C can be determined by considering only the linear term in (25): It can be determined by the conditions

$$a = 1 \qquad b < 0. \tag{28}$$

In the vicinity of the second-order phase transition line, the sublattice magnetization σ is given by

$$\sigma^2 = \frac{1 - a}{b}. \tag{29}$$

The right-hand side must be positive. If this is not the case, the transition is of the first order, and hence the point at which

$$a = 1 \qquad b = 0 \tag{30}$$

is the tricritical point. However, examining the numerical values of a and b in the system for all values of D, p and α defined by

$$\alpha = J'/J \tag{31}$$

one can find that the condition (30) is not satisfied in the two-dimensional decorated system with $S = 1$.

Figure 3 shows the variations in the transition temperature T_C determined from (28) as a function of D/J for the two systems with $p = 1.0$ and $p = 0.5$, selecting the five values of α. The solid curves labelled $p = 1.0$ and 0.5 are the same as the corresponding curves obtained in figure 2(a). Accordingly, in order that the compensation point (or points) may exist in the system with the fixed values of p and α, each solid curve a–j must be higher than the solid curves labelled $p = 1.0$ and 0.5. For instance, the system with $p = 1.0$ and $\alpha = 2.87$ can exhibit one compensation point in the region $-1.0 \leqslant D/J \leqslant 0.0$ and two compensation points in the region $-1.87 < D/J < -1.0$. The system with $p = 0.5$

and $\alpha = 2.87$ exhibits one compensation point for the region $D/J > -1.0$. In this way, comparing the behaviour of T_C in the figure with the results of figure 2(a), one can obtain much information on the behaviour of T_{COMP} in the system, although the system with $\alpha = 0.0$ (see curves i and j) does not exhibit any compensation point for any values of p.

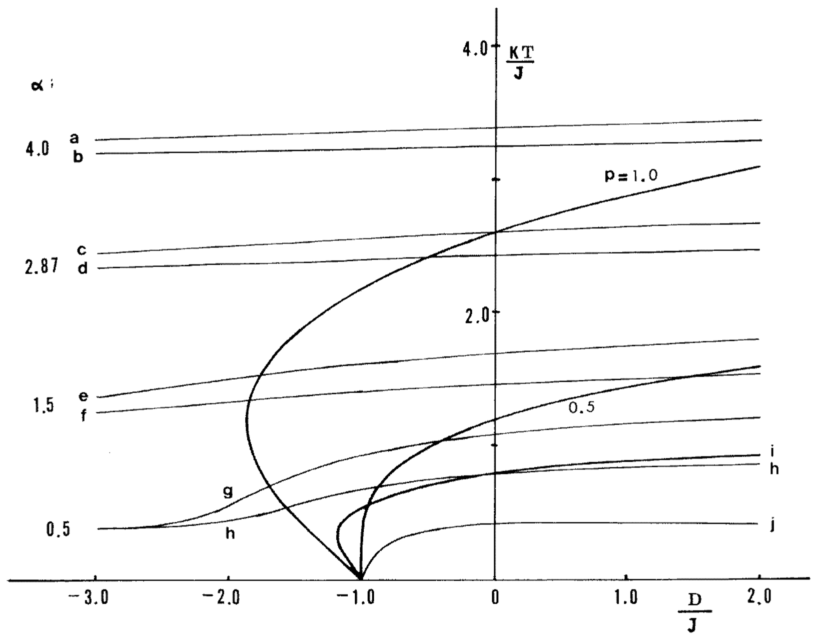

Figure 3. The phase diagram in the T–D plane for the decorated ferrimagnetic system depicted in figure 1, when the value of S is fixed at 1. The curves labelled $p = 1.0$ and 0.5 correspond to the results obtained from the relation $1 = 4pF_s(J)$ in figure 2(a). Curves a–j represent T_C. Curve a (for $p = 1.0$) and curve b (for $p = 0.5$) are obtained for the system with $\alpha = 4.0$. Curve c (for $p = 1.0$) and curve d (for $p = 0.5$) are for the system with $\alpha = 2.87$. Curve e (for $p = 1.0$) and curve f (for $p = 0.5$) are for the system with $\alpha = 1.5$. Curve g (for $p = 1.0$) and curve h (for $p = 0.5$) are for the system with $\alpha = 0.5$. Curve i (for $p = 1.0$) and curve j (for $p = 0.5$) are for the system with $\alpha = 0.0$.

In figure 4, therefore, the concentration dependences of T_C and T_{COMP} are depicted for the system with $\alpha = 2.87$, selecting the typical values of D/J from figure 3. As is seen from the figure, the T_C versus p curve does not exhibit a large variation for the change in D/J, when the value of J' (or α) takes a value higher than $\alpha = 0.5$, such as $\alpha = 2.87$. On the other hand, the T_{COMP} versus p curve in the system may exhibit a characteristic feature when the value of D/J becomes smaller than -1.0. As shown for $D/J = -1.5$, the system shows two compensation points in the region $0.8 < p \leqslant 1.0$ although, for the systems with $D/J = 0.0$ and $D/J = -1.0$, only one compensation point can be obtained in the regions $0.25 \leqslant p \leqslant 1.0$ and $0.5 \leqslant p \leqslant 1.0$, respectively. Here, note that these results are consistent with those predicted in figure 3.

Now, in order to prove whether the predictions of T_{COMP} obtained from figures 2(a), 3 and 4 in the decorated two-dimensional ferrimagnetic system with $S = 1$ are correct or not, it is necessary to study the temperature dependence of the total magnetization (6) or (10). The thermal variation in M in the system with $z = 4$ can also be obtained by solving

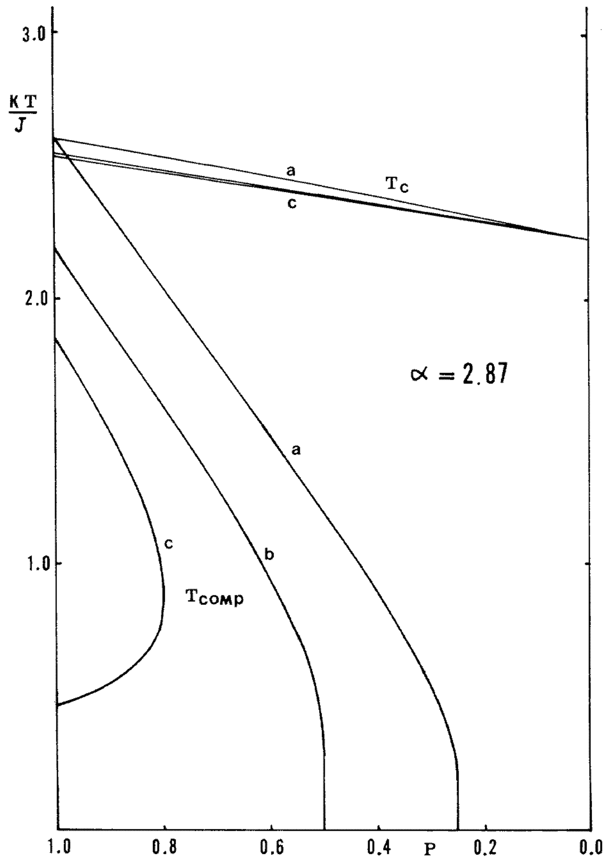

Figure 4. The phase diagram (T_C and T_{COMP}) in T–p space for the system with $\alpha = 2.87$, when three values of D/J are selected: curve a $D/J = 0.0$; curve b, $D/J = -1.0$; curve c, $D/J = -1.5$.

the coupled equations for σ, m and q of section 2 numerically. The numerical results of the system with $\alpha = 2.87$ are presented in figure 5, selecting typical values of p and D/J from figure 4. Curves a, b and c correspond to the thermal variations in M in the system with $p = 0.5$ when the three values of D/J are selected as 0.0, -1.0 and -1.5. These results are equivalent to the predictions of T_{COMP} obtained from figure 4. Curve a may exhibit one compensation point at a finite temperature but curves b and d do not exhibit any compensation point at a finite temperature, while curve c has a characteristic behaviour showing a minimum and a maximum below T_C. Curve d, which is obtained for the system with $p = 0.9$ and $D/J = -1.5$, may have two compensation points at the same temperature as those predicted from figure 4. Thus, one can understand that a variety of features for T_{COMP} in the decorated system with $S = 1$ is obtained correctly from figures 2(a), 3 and 4.

Finally, it is worth noting that the saturation magnetization of M in figure 5 is given by $|M|/N = 0.5$ for the curves a, c and d, although for curve b it reduces to zero. The reasons are as follows. Curve a is obtained for $D/J = 0.0$ and $p = 0.5$, so that the sublattice magnetizations σ and m at $T = 0$ K are given by $\sigma = 1/2$ and $m = -1.0$, and hence $|M|/N = 0.5$. On the other hand, for the systems with $D/J < -1.0$ the spin state of the

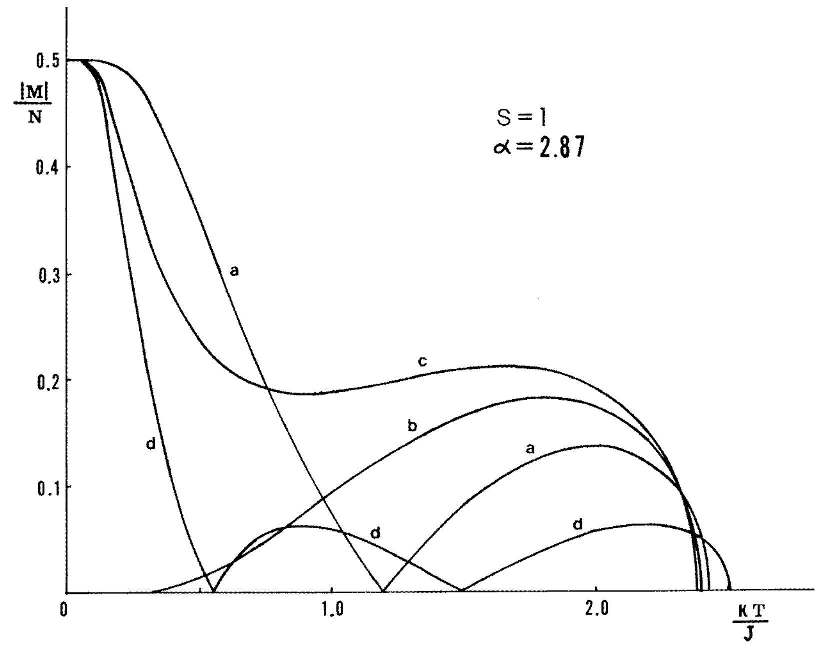

Figure 5. The temperature dependence of the total magnetization M plotted for the two-dimensional ferrimagnetic system with $S = 1$ and $\alpha = 2.87$, when four pairs of values of D/J and p are selected: curve a, $D/J = 0.0$, $p = 0.5$; curve b, $D/J = -1.0$, $p = 0.5$; curve c, $D/J = -1.5$, $p = 0.5$; curve d, $D/J = -1.5$, $p = 0.9$.

decorated B atoms is $S_m^z = 0.0$ at $T = 0$ K, independent of p, from which the saturation magnetization of curves c and d is also given by $M/J = 0.5$. However, the value of $D/J = -1.0$ is marginal, and hence half of the decorated magnetic B atoms are in the $S_m^z = -1.0$ state but the other half are in the $S_m^z = 0.0$ state at $T = 0$ K. From this, the saturation magnetization M for the system with $D/J = -1.0$ and $p = 0.5$ reduces to zero, since $\sigma = 1/2$ and $m = -1/2$ at $T = 0$ K.

3.2. The decorated system with S = 3/2

As discussed in section 3.1, the curves in figure 2(a) give us some indication of the compensation point (or points) in the system with $S = 1$, when the transition temperature T_C is larger than the corresponding curve in figure 2. As predicted in figure 2(b), on the other hand, the behaviour of equation (13) obtained for the decorated ferrimagnetic system with $S = 3/2$ is rather different from that of figure 2(a). It implies that the features of T_{COMP} in the system with $S = 3/2$ may be rather different from those discussed in section 3.1.

Let us here study the phase diagram (T_C and T_{COMP}) of the two-dimensional decorated system with $S = 3/2$ and $z = 4$. Following the same procedure as in section 3.1, we can also obtain an equation similar to (25) or (29). Then, for any values of D, p and α, one can find that the condition (28) is always satisfied for the two-dimensional system with $S = 3/2$ and hence the phase transition is also second order.

Figure 6 shows the phase diagram in T–D space for the two-dimensional decorated system with $S = 3/2$, when three values of α(= 3.5, 1.5 and 0.0) and three values of

p (= 1.0, 0.5 and 0.4) are selected. The thin and thick solid lines represent T_C and equation (13), respectively (or the corresponding results in figure 2(b)). Therefore, in order for a compensation point (or points) to exist in the system with fixed values of α and p, the T_C line must be higher than the corresponding thick line. For the case of $\alpha = 1.5$, a compensation point can be found in the system with $p = 1.0$ for the region $D/J < -2.02$ and in the system with $p = 0.5$ for the region $-1.0 < D/J < -0.76$. However, two compensation points can be obtained for the system with $\alpha = 1.5$ and $p = 0.4$ in the region $-0.79 < D/J < -0.5$. Thus, comparing the results of figure 6 with those of figure 3 or figure 4, one can find that the roles of p for finding the two compensation points are rather different, depending on whether the value of S in the system is 1 or 3/2.

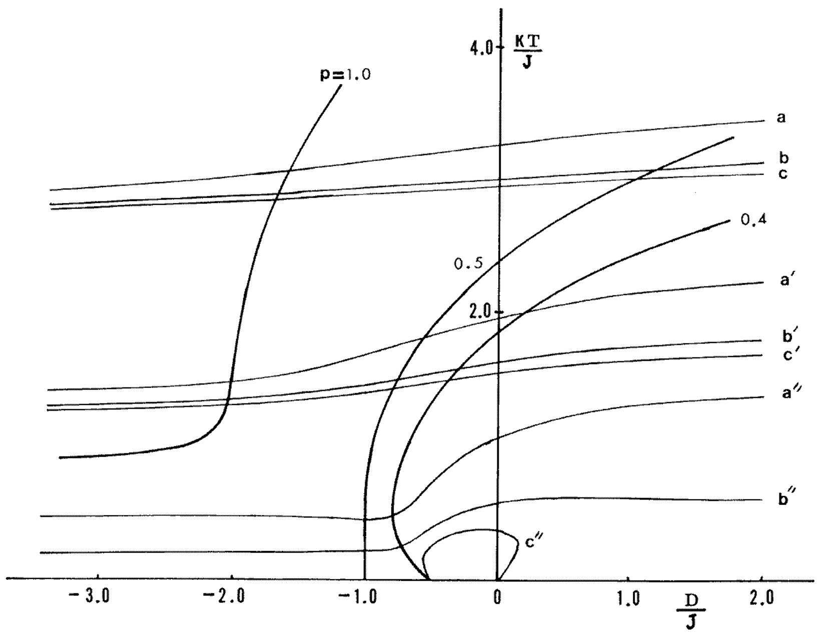

Figure 6. The phase diagram in the T–D plane for the decorated ferrimagnetic system, when the value of S is fixed at 3/2. The thick solid lines labelled $p = 1.0$, 0.5 and 0.4 are the results obtained from equation (13) and the thin solid lines represent T_C. Curves a, b and c are obtained for the system with $\alpha = 3.5$, selecting three values of p (curve a, $p = 1.0$; curve b, $p = 0.5$; curve c, $p = 0.4$). Curves a′, b′ and c′ are for the system with $\alpha = 1.5$ and curves a″, b″ and c″ are for the system with $\alpha = 0.0$, when three values of p are selected as (curves a′ and a″, $p = 1.0$; curves b′ and b″, $p = 0.5$; curves c′ and c″, $p = 0.4$).

In figure 7, the phase diagram of the system with $S = 3/2$ and $\alpha = 1.5$ is depicted in the T–p space, changing the value of D/J. The thin and thick solid lines represent T_C and T_{COMP}, respectively. The figure clearly indicates that the role of p for finding the two compensation points in the decorated system with $S = 3/2$ is different from that in figure 4. The possibility of two compensation points in the system with $S = 3/2$ and $\alpha = 1.5$ can be found in the region $0.355 < p < 0.5$ when $D/J = -0.7$. In figure 4, on the other hand, the possibility of two compensation points in the system with $S = 1.0$, $\alpha = 2.87$ and $D/J = -1.5$ is found even for the pure ($p = 1.0$) case but disappears for a concentration less than $p = 0.8$. Thus, for the system with $S = 3/2$ the possibility of two compensation points can be obtained only when there is a large dilution of non-magnetic atoms, namely

$0.2 < p < 0.5$, in the decorated sites (or full circles in figure 1), depending on the values of a negative D/J and a finite value of α.

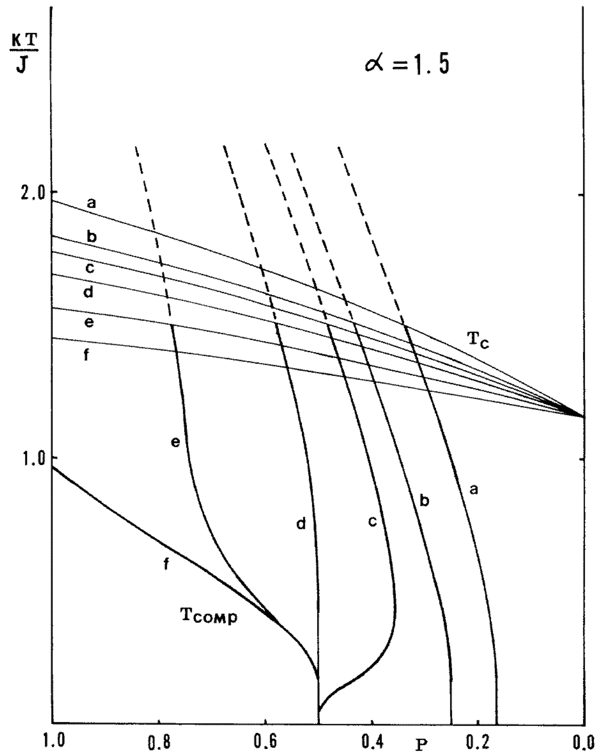

Figure 7. The phase diagram (T_C and T_{COMP}) in T–p space for the system with $\alpha = 1.5$, when six values of D/J are selected: curve a, $D/J = 0.0$; curve b, $D/J = -0.5$; curve c, $D/J = -0.7$; curve d, $D/J = -1.0$; curve e, $D/J = -1.5$; curve f, $D/J = -2.5$.

4. Conclusions

In this work, we have studied the magnetic properties (T_C, T_{COMP} and magnetization curve) of the decorated ferrimagnetic Ising system with three kinds of atom, of which one with spin 1/2 forms the sublattice L_1 and the other two (a spin-S ($> 1/2$) magnetic atom and a non-magnetic atom) are randomly distributed on the sublattice L_2. They have been discussed within the framework of the EFT with correlations. Numerical results are obtained in section 3 for the two-dimensional lattice. In particular, we have examined in detail the effects of crystal-field interaction and spin value in the spin-S atoms on T_C and T_{COMP}. As shown in figures 2–7, the results obtained are extremely interesting.

In this work, we have applied the EFT to numerical evaluations of σ and T_C for the two-dimensional system with $S = 1$ or $S = 3/2$. However, one should note the following facts. If the exact calculation can be made for a system with a finite (negative) value of D, the exact T_C will be lower than that obtained from the EFT, as discussed for the mixed spin-1/2 and spin-S Ising system on the honeycomb lattice [15] having a Hamiltonian similar to (1). On the other hand, the evaluation of T_{COMP} in the decorated ferrimagnetic Ising system can be done exactly as noted in sections 2 and 3. The situation for finding the compensation point (or points) in the decorated system with S does not change even when the exact T_C is obtained, since the T_{COMP} lines in figures 3–7 are the exact results. Then, the only difference is that the critical value of α and p for finding T_{COMP} will be changed in the phase diagrams.

Finally, we have shown in this paper that the decorated ferrimagnetic Ising system with magnetic and non-magnetic atoms randomly distributed on the sublattice L_2 may exhibit many unexpected features in the phase diagrams, depending on the values of D, p and S. The possibilities of two compensation points are clearly different in the systems with $S = 1$ or $S = 3/2$ and the thermal variation in M for curve c in figure 5 exhibited a maximum and a minimum below T_C. These results have not been predicted in the Néel theory of ferrimagnetism [1, 2]. Thus, we may conclude by saying that the decorated ferrimagnetic Ising system investigated here is a fruitful system from both the theoretical and the materials science point of view. We hope that the present study will stimulate experimental and theoretical work on the systems considered here.

Appendix.

The functions $G_s(x)$ and $R_s(x)$ in (24) are defined by

$$G_s(x) = \frac{2\cosh(\beta x)}{2\cosh(\beta x) + \exp(-\beta D)} \qquad \text{for } S = 1 \tag{A1}$$

and

$$G_s(x) = \frac{1}{4}\frac{9\cosh(3\beta x/2) + \exp(-2\beta D)\cosh(\beta x/2)}{\cosh(3\beta x/2) + \exp(-2\beta D)\cosh(\beta x/2)} \tag{A2}$$

$$R_s(x) = \frac{1}{8}\frac{27\sinh(3\beta x/2) + \exp(-2\beta D)\sinh(\beta x/2)}{\cosh(3\beta x/2) + \exp(-2\beta D)\cosh(\beta x/2)} \qquad \text{for } S = 3/2. \tag{A3}$$

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